Int. J. Heat Mass Transfer. Vol. 13, pp. 1115–1131. Pergamon Press 1970. Printed in Great Britain

ON THE POSSIBILITY OF MEASURING THERMAL FIELDS USING THE MÖSSBAUER EFFECT– APPLICATION TO TURBINE BLADING

FRANCESCO P. BRANCA and PAOLO LANCIANI
Istituto di Macchine e Tecnologie Meccaniche dell'Università di Roma, Roma, Italy

(*Received* 6 *March* 1969 *and in revised form* 6 *January* 1970)

Abstract— The use of the Mössbauer effect to measure thermal gradients inside solid samples is proposed and described. The gamma rays produced in a Mössbauer source are absorbed by active nuclei seeded in the sample under observation. It is then possible, by analysing the data relative to the absorption intensity, to obtain the values of both the strongly temperature-dependent recoil-less fraction and second-order Doppler shift. The knowledge of the mean temperature value along the penetration direction can be reached with fair accuracy and then by repeating the measurement, the thermal gradients inside the sample.

Moreover, the application of this method to the study of thermal fields in turbine blading is proposed, and some rough estimates of the source intensity, resolving power and time intervals needed are given.

NOMENCLATURE

a, resolving power;
a_0, lattice crystal parameter;
A, source activity;
c, light velocity in vacuum;
C, counting rate;
E, $= h\nu$;
E_0, $= h\nu_0$;
ΔE, energy shift;
f, Mössbauer events fraction;
h, Planck constant;
$\hbar$, $= h/2\pi$
k, Boltzmann constant;
l, thickness of the sample;
m, mass of emitting nucleus;
n, number of counts;
r, length of the sample;
s, relative velocity between source and absorber;
s_i, isomeric shift;
s_d, second order Doppler shift;
s_d^{class}, leading term of expansion of s_d;
T, absolute temperature;
v, nucleus velocity before emission;
v', nucleus velocity after emission;
$\langle v^2 \rangle$, mean square velocity;
$2W$, Debye–Waller factor;
x, material thickness;
z, unit vector in the z direction;
α, crystal lattice stiffness;
α', restoring forces acting on the emitting atom;
ε, statistical error on n;
Γ, linewidth;
μ, mass absorption coefficient;
η, detector efficiency;
ν, actual frequency of emitted gamma-ray;
ν_0, expected frequency of emitted gamma-ray;
$\Delta\nu^{rec}$, frequency shift caused by nucleus recoil;
σ_0, resonance absorption cross-section;
σ, Mössbauer self absorption cross-section;
τ, lifetime of excited nucleus;
ρ, ϑ, polar coordinates.

Subscripts

a, absorbing nucleus;
e, emitting nucleus.

1. INTRODUCTION

THE POSSIBILITY of measuring with good accuracy thermal fields existing inside metallic samples seems to the authors to be most useful in many problems arising in heat exchange studies. To the authors' knowledge, no procedure has been proposed until now for the measurement of local temperature within solid mediums without any appreciable interaction on their thermal, mechanical or electric state.

The procedure that can be used for this purpose is the strongly temperature-dependent recoil-less resonant absorption of nuclear gamma rays by nuclei bound in a crystal lattice [1]. This effect, discovered by Mössbauer in 1957, has been widely used since then in numerous application [2, 3] but, although Mössbauer analysers and spectrometers can be easily found, being already produced by numerous manufacturers,* it appears to the authors that the use of this effect for the measurement of the temperature inside solid samples, and of the thermal gradients existing herein, has not yet been actuated or proposed. In fact only the Mössbauer effect temperature-dependence and its availability for temperature measurement has been mentioned by Gonser [4].

With the use of the Mössbauer effect, which will be described in the next section, the temperature can be obtained in the following way: some nuclei of the sample, in which the thermal gradients have to be measured, absorb, along a fixed direction, a fraction of a gamma-ray beam emitted by a moving source. The absorbed fraction along the beam direction depends on the sample temperature, and on the relative velocity between source and sample; moreover, the relative velocity for which this absorbed fraction is maximum depends on the temperature difference between source and sample. These characteristics of the Mössbauer effect allow to determine the sample mean temperature along the beam direction.

When the sample is scanned by the beam along different directions, it is possible to obtain the thermal gradients orthogonal to the beam directions.

In this paper, after a brief survey of the analysis of the Mössbauer effect, and the experimental arrangements and results found useful in this case, the application of the method to the study of turbine blading will be proposed.

2. THE MÖSSBAUER EFFECT AND ITS TEMPERATURE DEPENDENCE

In order to explain the temperature dependence of the Mössbauer effect, a brief account of the behaviour of emission, absorption and scattering of photons by atomic nuclei is needed. When an atom of mass m emits a photon of frequency ν, energy and momentum conservation laws hold and may be written in the following form [5]

$$h\nu = h\nu_0 + \tfrac{1}{2}m(v^2 - v'^2) \tag{1}$$

$$m\vec{v}' = m\vec{v} - \frac{h\nu}{c}\vec{Z} \tag{2}$$

where $\vec{v}$ and $\vec{v}'$ are the velocities of the centre of mass before and after photon emission, h is the Planck constant, $h\nu_0$ the change in internal energy of the atom, and $\vec{z}$ a unit vector in the direction of emission.

Squaring (2) and substituting in (1) after having replaced ν by ν_0 (which is permissible because these terms are very small compared with $h\nu_0$), the result is

$$\frac{\Delta\nu}{\nu_0} = \frac{\nu - \nu_0}{\nu_0} = \frac{\vec{z}\cdot\vec{v}}{c} - \frac{h\nu_0}{2mc^2}. \tag{3}$$

Equation (3) expresses the frequency shift of the photon. The first term on the right expresses the usual Doppler shift due to the fact that the emitting atom is not at rest; the second term expresses the recoil shift due to the fact that the atom has a finite mass.

Now, when an ensemble of free atoms is considered, equation (3) remains valid for any single emitted photon. However, emitted pho-

* For example by Nuclear Science and Engineering Corp, Pittsburgh; Carl Zeiss, Jena; etc. . . .

tons give rise to an emission line which, by the Heisenberg uncertainty principle, has a natural width due to the energy (or frequency) uncertainty between emitted photons. The shape of this line is Lorentzian, and its width is proportional to $1/\tau$ where τ is the lifetime of the excited state of the emitting nuclei. In this case, the first term on the right of equation (3) gives rise, because of the velocity distribution of the nuclei due to thermal motion, to a broadening of the line (Doppler linewidth), which assumes a Gaussian shape. The second term of equation (3), which gives rises to a line shift, becomes significant only in the case of gamma ray emission by nuclei, because in this case the corresponding line width is sufficiently small.

Consequently, the emission line of an ensemble of atoms is broadened by the thermal motion Doppler effect, and is shifted by the amount:

$$\Delta\nu_{\text{rec}} = \frac{h\nu_0}{2mc^2}.$$

The same situation obviously holds in the absorption case, with a shift of the line in the opposite direction (increased frequency).

In these conditions, and when the recoil shifts are significant, resonance absorption, that is absorption of emitted photons, cannot occur unless Doppler broadening is sufficient to make parts of the two spectra overlap.

Mössbauer discovered that, when the nuclei are bound in a solid lattice, a certain fraction of photons are emitted or absorbed without recoil shift and Doppler broadening of the linewidth, this fact making a large resonance absorption possible. This is explained, in analogy with the theory of neutron absorption resonance proposed by Lamb [6], if energy and momentum conservation laws hold with the crystal as a whole, thus taking up the differences, and making the recoil energy and velocity negligible when averaged over all the nuclei. These recoil-free gamma rays, emitted in zero-phonon processes, leave the crystal in the same quantum state as before the interaction.

On the contrary, the other gamma rays are emitted with simultaneous emission or absorption of phonon by the solid lattice, and hence with an energy difference.

This effect is very useful in numerous applications because the natural width of the nuclear transitions is very small, compared to that of the X-rays emitted in orbital electron transitions, as their lifetime τ is very much longer (10^{-7} s for ^{57}Fe against 10^{-14} s for a similar energy X-ray). This fact ensures that the resonance absorption is very sharp and any factor that changes either the energy of the emitted gamma ray or the level spacing of the absorber nuclei by a very small amount (about 10^{-9} eV) is sufficient to prevent it. The Mössbauer effect is thus a tool for studying all the causes that can vary the energy levels in the crystal.

The fraction of photons emitted or absorbed in zero-phonon (or recoil-less) transitions is strongly temperature-dependent, as is also another effect occurring, the second-order Doppler shift, which can be detected by Mössbauer spectrometers. A brief survey on the analysis of these two temperature-dependent characteristics will be made in the rest of this section.

(a) *The recoil-free fraction*

The recoil-less fraction was considered first by Mössbauer in his original work [1] and later analysed by numerous other workers [7–9].

To evaluate the self-absorption cross-section, which is defined as the probability that a recoil-less photon be first emitted in a source, and afterwards absorbed in another recoil-less transition in an absorber, a description of an actual crystal is needed. This evaluation has been done for simple models and, in particular, for a cubic Bravais lattice (that is a lattice with cubic unit cells containing one atom per cell) in the harmonic approximation; in this approximation, the potential describing the atomic forces is expanded in a power series, and reduced to terms of second order. This cross-section is given by the expression [9]:

$$\sigma(s) = \frac{\pi\sigma_0\Gamma}{2}\exp\{-(2W_e + 2W_a)] \times \left[\frac{\Gamma}{\pi}\frac{1}{s^2+\Gamma^2} + \sum_{n=1}^{\infty}\frac{(2W_e + 2W_a)}{n!} f_n(s)\right] \quad (4)$$

where s is the relative velocity between emitter and absorber (positive if they move towards each other), $2W_e$ and $2W_a$ are the Debye–Waller factors, respectively for the emitter and the absorber, Γ the natural width of the excited state of the nucleus, and σ_0 the resonance absorption cross-section for the absorbing nucleus.

The second term in equation (4) corresponds to transitions with phonon exchange in the crystal; while the first, corresponding to the zero-phonon process, gives the required fraction of recoil-free events. The self-absorption cross-section for zero-phonon processes is then given by:

$$\sigma(s) = \tfrac{1}{2}\sigma_0\Gamma^2\frac{1}{s^2+\Gamma^2}\exp[-(2W_e + 2W_a)]. \quad (5)$$

It has been shown that equation (5), which could be reached also by other general theoretical considerations [9], is correct to a very close approximation even when anharmonicity is taken into account [10].

The Debye–Waller factor temperature-dependence has been studied extensively, and calculated numerically for several different models of crystals. In the following, only the value of this function for the very important case of an emitting or absorbing impurity atom will be reported. In fact, in the more usual case which could be of interest in the proposed application of the Mössbauer effect measurement for the study of thermal gradients, the resonant nuclei will be impurity atoms (for example ^{57}Fe) seeded, homogeneously or not, in the sample under consideration.

Numerical calculations of the Debye–Waller factor temperature dependance for several models of host crystals and different impurity atoms have been carried out [11–14].

In the case where the impurity atom is roughly the same size as the host lattice atoms, and using the nearest neighbour central force approximation [15] for evaluating the force constant changes, Maradudin and Flinn [11] have written the Debye–Waller factor for a face-centred cubic crystal in the high-temperature limit in this form:

$$2W = 0{\cdot}420\frac{E^2}{\hbar^2c^2}\frac{kT}{\alpha}\left[1 + 0{\cdot}596\left(\frac{\alpha-\alpha'}{\alpha}\right) + 0{\cdot}744\left(\frac{\alpha-\alpha'}{\alpha}\right)^2 + \ldots\right] \quad (6)$$

where α is a measure of the stiffness of the crystal lattice (in this approximation, the bulk modulus is equal to $\frac{4}{3}\alpha/a_0$, where a_0 is the lattice parameter) and α' represents the restoring forces acting on the impurity atom.

When $\alpha' \simeq \alpha$, the recoil-less events fraction can be written in the final form:

$$\sigma'(s) = \tfrac{1}{2}\sigma_0\Gamma^2\frac{1}{s^2+\Gamma^2} \times \exp\left(-0{\cdot}420\frac{E^2}{\hbar^2c^2}k\left[\frac{T_e}{\alpha_e} + \frac{T_a}{\alpha_a}\right]\right). \quad (7)$$

Equation (7) shows that the self-absorption cross-section for nuclear gamma rays vary greatly with temperature. This result is generally in very good agreement with the experimental data, as will be shown in section 3. This fact makes Mössbauer effect intensity suitable for the measurement of temperature.

(b) *The second-order Doppler shift*

The analysis carried out in the preceding section is not strictly exact, as it does not predict that the centre of the recoil-less peak is slightly shifted in frequency from the condition $E = E_0$ (or $s = 0$, being $s/c = (E - E_0)/E$ because of the Doppler shift, due to relative motion between emitter and absorber).

In the absence of electric field gradients and magnetic fields, two shifts of the resonance peak are superimposed:

(1) The isomeric shift, which arises from the electrostatic interaction between the nuclear charge distribution (different in the ground and the excited states), and the electronic charge density at the nucleus, which changes the separation of the levels [16]; this shift may be regarded usually as independent of temperature [17] and can be separated from the second order Doppler shift, which is interesting for our purposes; and

(2) The second order Doppler shift, which is a relativistic effect due to the fact that the mass of the ground state of the nucleus differs from that of the excited state.*

The energy changes due to isomeric shift and second-order Doppler shift can be expressed in terms of the relative velocity s between source and absorber, using the first-order Doppler shift expression

$$\frac{\Delta E}{E} = \frac{s}{c}. \tag{8}$$

The measured total shift of the Mössbauer peak will be given by:

$$s = s_{iu} + s_{du} - s_{ie} - s_{de} \tag{9}$$

where s_{ia} and s_{ie} are respectively the isomeric shift of absorber and emitter, and s_{du} and s_{de} their second-order Doppler shift.

It has been shown by Josephson [18] that, in the high temperature-limit (i.e. for temperature greater than the characteristic Debye temperature of the material being considered) the second-order Doppler shift can be expanded in inverse powers of temperature, and that the leading term of this expansion is equal to the classical value:

$$s_d^{\text{class}} = \frac{3kT}{2mc}. \tag{10}$$

This result is a very general one, since it has been shown that anharmonicity, presence of impurities, distortions in the crystal lattice affect only the other terms, but not the leading term (10) of s_d. With this value, equation (9) becomes:

$$s = (s_{iu} - s_{ie}) + \frac{3k}{2c}\left(\frac{T_a}{M_a} - \frac{T_e}{M_e}\right). \tag{11}$$

Equation (11) shows that if the source is maintained at a constant temperature, the measured Mössbauer peak shift is directly proportional to the absorber temperature. This result has been very well fitted by experimental data, as will be recalled in the next section.

3. EXPERIMENTAL DEVICES AND RESULTS

Equation (7) of the preceding section gives the probability that a gamma-ray, emitted in a recoil-less transition, will be absorbed by a nucleus bound in a lattice in another recoil-less transition. It is evident from this equation that such probability increases when the energy of the gamma-ray is low, the material in which the active nuclei are bound stiff, and the temperature decreases.

The Mössbauer effect has been observed up to now for a great number of nuclei [20], but only a few are usable as tools in general applications.

In fact, only a few nuclei have nuclear transitions with an energy difference sufficiently low to be observed, about 10–20 keV, as can be seen from equation (7), at temperatures higher

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* In fact, when the nucleus returns to the ground state, it emits a gamma-ray (of energy E); the corresponding mass difference is:

$$\Delta m = \frac{E}{c^2}$$

This decrease of the nucleus mass causes an increase in its kinetic energy after the emission has occurred. This energy must be returned to the nucleus and, as a consequence, the energy of the emitted gamma-ray is correspondingly decreased by the same amount. If $\langle v_e^2 \rangle$ is the mean square velocity of the emitting atom, this amount is [18, 19]:

$$\Delta E = \tfrac{1}{2}\Delta m \langle v_e^2 \rangle = \tfrac{1}{2} E \frac{\langle v_e^2 \rangle}{c^2}.$$

Obviously, the same effect occurs in the absorption case, so that any experiment will measure only the difference $\langle v_e^2 \rangle - \langle v_a^2 \rangle$ [17] where $\langle v_a^2 \rangle$ is the absorbing atom mean-square velocity; it is this difference that depends on temperature.

than room temperature (nuclear transitions are in general in the order of MeV).

Moreover, in addition to the conditions expressed by equation (7), the Mössbauer effect can be observed only if some means of access to the first excited level of the emitting nucleus exists. Only radioactive decay is at present suitable, since fluorescent excitation by X-rays, for example, is impracticable, because the fraction of the obtained radiation lying in the resonant band of the nucleus is too small. A suitable radioactive parent is not always available for the observation of the effect [3, 20].

Another difficulty occurs when the emission (or absorption) line is too sharp (for example, in the case of ^{67}Zn and ^{182}W), because in this case the resonance is too dependent on external conditions to be maintained over the time needed for the measurements.

In conclusion, at the present time, only two nuclei seem truly suitable for such engineering application as proposed in this paper, namely ^{57}Fe and ^{119}Sn [3]. In Table 1, the characteristics of the Mössbauer effect that can be obtained with these two isotopes are summarized.

The instrumentation for the measurement of the Mössbauer effect is composed of the following parts, arranged as shown in Fig. 1:

A. A source emitting the recoil-less gamma rays;
B. A sample that can absorb the emitted recoil-less gamma rays in another recoil-less transition;
C. A gamma-ray detector suitable to the energy of the photons being considered;
D. An electronic apparatus to count the detector signals corresponding to the gamma rays detected;
E. A suitable device to give some motion to the source (or to the absorber B), in order that the relative velocity s between source and absorber may at any time be known with very great precision. This relative velocity, by means of the first order Doppler effect, gives an energy difference between source and emitter atoms as shown in equation (8); it is then possible to vary this energy difference in a continuous way by varying slightly the value of s.

When the energy difference due to the relative velocity reaches a value that exactly balances the energy shifts existing between emission and absorption (isomeric shift and second-order Doppler shift in the case considered, as seen in the preceding section), Mössbauer resonance absorption occurs, giving rise to an absorption peak in the number of gamma rays that reach the detector C and are counted.

A typical absorption peak is shown in Fig. 2 for a 2mC ^{57}Co source and an absorber made of $K_4Fe(CN)_6 \cdot 3H_2O$ containing 5 mg/cm^2 Fe.

The relative velocities s needed are in the range 0–3 cm/s, being the associated energy differences $\Delta E/E$, given by equation (9), in the range $0 - \sim 10^{-10}$.

Obviously, the difficulties in a Mössbauer instrumentation lies in the great accuracy

Table 1. ^{57}Fe and ^{119}Sn Mössbauer transition characteristics

	^{57}Fe	^{119}Sn
Radioactive parent	$^{57}Co_{27}$	$^{119}Sn_{50}$
Radioactive process for excited level populating	Electron capture	Isomeric transition
Parent lifetime, d	270	245
Mössbauer gamma energy, keV	14·4	23·8
Excited level lifetime, 10^{-8} s	9·77	1·84
Mössbauer active isotope in natural material, %	2·19	8·58
Isotopically enriched material usually available, %	>90	>85

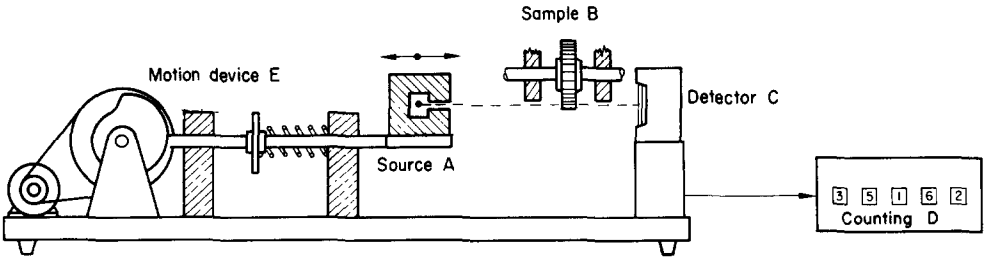

FIG. 1. Sketch of a Mössbauer effect typical analyser.

needed in the relative velocity control. The existing instrumentation can be substantially divided into two classes [21]:

(a) Constant-velocity, or Mössbauer analysers, where the relative velocity is maintained constant, and a gamma-ray intensity is obtained successively for any required velocity; with an instrument of this kind, a spectrum of the type of Fig. 2 is obtained.

(b) Variable-velocity. or Mössbauer spectrometers, where the velocity, not being constant, is measured at every instant; in this case, a multichannel counter, with an associated electronic circuitry, correlates at each instant the detector counts with the actual velocity, and records them until a data display is required [22]. A spectrum as shown in Fig. 3 is obtained in this case.

Some instruments exist also that can function with both characteristics, according to experimental needs [21, 23].

As already said, the results obtained, in

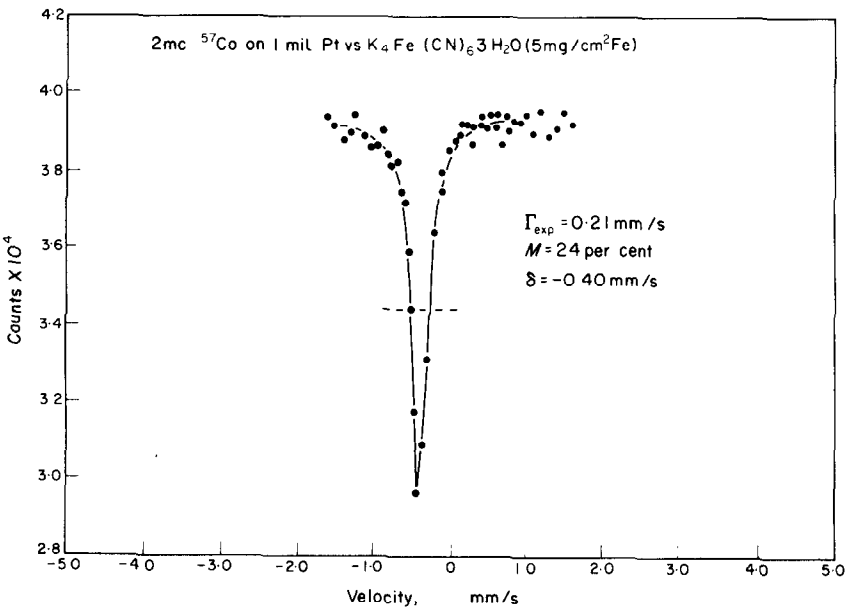

FIG. 2. Mössbauer absorption peak, as measured by a typical analyser with constant velocity.

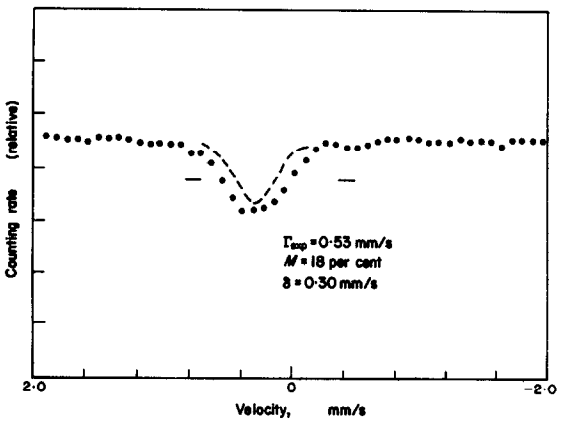

FIG. 3. Mössbauer absorption peak, as measured by a typical spectrometer with constant acceleration.

several experiments for the study of the Mössbauer-effect temperature behaviour carried out in the last few years, are in very good agreement with the theory sketched in the preceding section.

Much data about the temperature dependence of the recoil-less fraction is available, and some of this, obtained with ^{57}Fe and ^{119}Sn embedded in different host materials, is summarized in Table 2. The experiments have been performed by different authors under very different conditions of temperature range, source and absorber preparation and thickness, and gamma-ray detectors.

Obviously, equation (7) has been obtained through many simplifications, the results are only approximate to a calculated curve, but if atomic force potential anharmonicity is taken into account [10], many small discrepancies can be explained [24, 25] It is obvious that these considerations are to a large extent unimportant in the case considered in this paper, since only relative values are needed and all the system would have to be accurately calibrated before any temperature measurement can be performed.

As an example, the characteristic behaviour of the recoil-less fraction in a platinum matrix, obtained for ^{57}Fe impurities by Steyert and Taylor [26] and for ^{119}Sn by Bryukhanov *et al.* [27] is shown in Fig. 4.

The errors quoted in column 5 of Table 2 are those which affect the absolute values of the recoil-less fraction measured against known absorbers; the relative changes are determined with much smaller errors [27].

4. THE MÖSSBAUER EFFECT FOR THE MEASUREMENT OF THERMAL FIELDS

It has been seen in the preceding sections that both the recoil-less fraction given by equation (7) and the second-order Doppler shift are temperature-dependent. Both are then suitable for the measurement of the thermal gradients inside a sample. Obviously, the measured recoil-less fraction depends, for a given source, not on the temperature, but also on the density of absorbing nuclei in the absorber, on the sample thickness along the gamma-ray beam direction and on the stress differences that can change the value of the interatomic forces within the sample.

These facts have the result that the reliability of this effect for the measurement of temperature is not very high.

A method that can be used to obtain the sample mean temperature along a fixed direction is to measure the recoil-less absorption fraction for different relative velocities between source and absorber, in order to have a resonance curve of the kind shown in Fig. 2. These curves, one for each chosen direction, must be fitted to a Lorentzian line shape, by means of a least-square analysis that yields the values of three parameters of the line (fractional depth and width of the absorption line and its position), and the probable errors affecting each of these quantities [25].

The knowledge of these parameters allows the use of both effects simultaneously; this possibility increases the validity of the method proposed.

Some considerations of an experimental nature have to be made at this point:

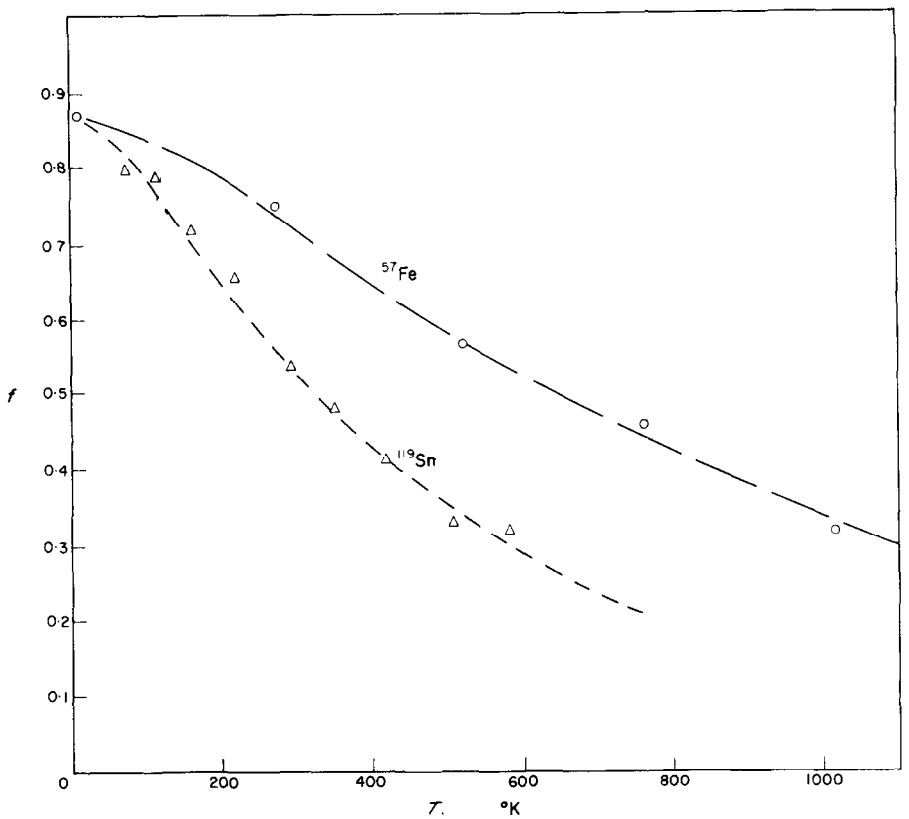

FIG. 4. The Mössbauer recoil-less fraction for ^{57}Fe and ^{119}Sn impurity atoms seeded in a platinum matrix.

Table 2. Summary of some experiments on Mössbauer temperature dependence

Mössbauer isotope	Matrix	Temperature field, °K	2° order Doppler shift	Error, %	Reference
^{57}Fe	Indium	4–420		5	24
^{57}Fe	Gold	77–700		8	14
^{57}Fe	Be	82, 298, 420	observed	0·5–1	25
^{57}Fe	Cu	82, 298, 420	observed	0·5–1	25
^{57}Fe	W	82, 298, 430	observed	0·5–1	25
^{57}Fe	Pt	82, 298, 425	observed	0·5–1	25
^{57}Fe	Au	4–1050	observed	1·5	26
^{57}Fe	Cu	4–1050	observed	1·5	26
^{57}Fe	Ir	4–1050	observed	2·5	26
^{57}Fe	Pd	4–1050	observed	1·5	26
^{57}Fe	Pt	4–1050	observed	1·5	26
^{57}Fe	Rh	4–1050	observed	1·5	26
^{57}Fe	Ti	4–500	observed	2·5	26
^{57}Fe	Fe	180–600	observed	1	17
^{57}Fe	Be	180–600	observed	1	17
^{119}Sn	Vanadium	77–660		10–15	13
^{119}Sn	Au	77–485		10	27
^{119}Sn	Pt	77–580		10	27
^{119}Sn	Thallium	77–220		10	27

1. The error in temperature will depend on the errors affecting the counting rates corresponding to each value of the relative velocity s between source and absorber. Since nuclear emission and absorption phenomena follow the Poisson statistics, the r.m.s. ε will be given by:
$$\varepsilon = \sqrt{[n(s)]} \qquad (12)$$
where $n(s)$ is the number of Mössbauer events corresponding to velocity s. $n(s)$ will be contained in general in the range 10000–40000 in order to obtain relative errors about respectively 1–0·5 per cent.
2. The resolving power for the determination of the temperature will depend on the collimation of the gamma-ray beam radiating from the source. This focalization, which is obtained by shielding the source [28], decreases its intensity.
3. The intensity of available Mössbauer sources is in the range 1–25 mC (1 mC corresponds to the emission of $3{\cdot}7 \times 10^7$ gamma rays/s); greater intensities will require precautions for one's personal safety [29].
4. The exigencies expressed in the preceding points can be each reconciled by increasing the time needed for the determination of each point in the spectrum relative to a direction of penetration of gamma rays inside the sample.
However, the contribution of the non-resonant background (caused principally by Compton scattering near the detector) to the total counting increases with the time interval needed to make the measurement.
5. The active thickness of the absorber is then determined, for a given source, by the need to obtain a measurable effect in a reasonable time; in general, thickness of a few mg/cm^2 is utilized. In the case of thermal gradient measurement inside samples of greater thickness (few cm), the Mössbauer active nuclei will have to be seeded in an inactive material.
6. Gamma rays of the energy considered (a few tenths of keV) interact very strongly, by photoelectric and Compton effects, with the material [30] penetrated. However, if the sample is made of beryllium, sufficient thicknesses can be utilized without too greatly decreasing the counting rates [30, 31]. Moreover, the mechanical properties of beryllium make it suitable also for dynamic measurement of thermal gradients in such devices as bearings, turbine blades, gears, etc. Some of the physical, mechanical and thermal properties of beryllium [31, 32] as compared with those of aluminium and iron, are reported in Table 3. Beryllium toxicity can be easily avoided if necessary precautions are taken in working and handling it [31].

These conditions can frequently be satisfied in useful and interesting applications as is shown in the following example.

In the study of the stresses induced in the walls of an horizontal pipe by a low-temperature fluid flowing in it at various levels [33, 34], it is important to measure the temperature distribution in a partially-filled annulus. The annulus can easily be made of beryllium and its thickness 1 can be of the order of a few cm. For a geometry of this kind (Fig. 5), it is possible, by varying the position of the annulus relative to the source-detector direction AC, to obtain a plot of the mean temperature along the z axis, for different values of ρ and θ, for different temperatures and levels of the liquid inside the annulus.

It is interesting to compare the values obtained in this manner with the results of Emery *et al.* [35], obtained by using 25 copper–constantan thermocouples cemented to the interior surfaces of the pipe.

5. MEASUREMENT OF HEAT TRANSFER TO TURBINE BLADING

The use of the Mössbauer effect could be essential in these engineering applications where measurement of thermal fields inside solid samples is very important, and the use of traditional methods indirect and difficult, A typical case is that of moving part of machines

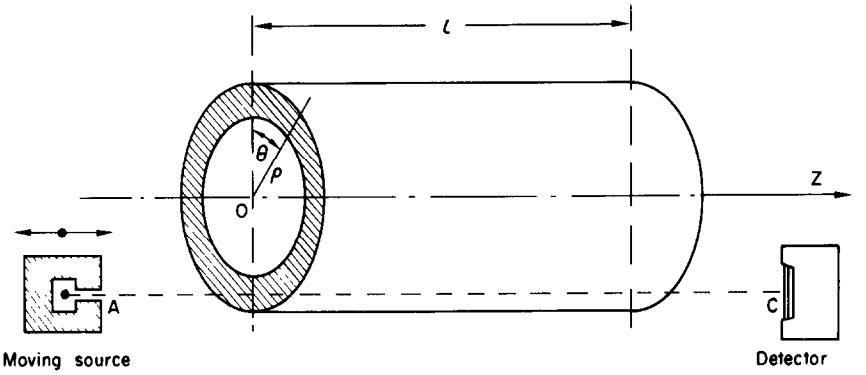

FIG. 5. Experimental set-up for thermal field measurement in a partially filled annulus.

where thermocouples and other conventional thermometers are inadequate because, when applied, and this is not always easy to do, they can disturb both the thermal fields to be measured and the stability of the machine during its motion.

An illuminating example, which will be discussed in some detail because of its importance, is the study of heat transfer to turbine blading. It is well known that, in order to increase turbine efficiency to the highest possible levels, the inlet flow temperature must be raised [36, 37]. The difficulties that arise when designing the cascade geometry and blade profile, and making a good selection of materials, require rather precise knowledge of the thermal gradients existing inside the blades. In fact, turbine blading are subject to mechanical stresses of thermal origin, besides those caused by centrifugal forces or by fluid action, when the turbine is moved by hot gases [38]. The amplitude of thermal stresses depends on the temperature differences between two points in the blade and reach its maximum value when the machine is started or stopped.

During recent years many workers have faced the problem of the measurement of thermal gradients within the blades. To obtain them, Wilson and Pope investigated the distribution of

Table 3. Some properties of beryllium, compared to aluminium and iron

	Beryllium	Aluminium	Iron
Atomic weight	9·013	26·97	55·85
Density (25°C), g/cm^3	1·84	2·7	7·87
Atomic volume, cm^3/mole	4·88	9·99	7·1
Crystal structure	hcp	fcc	bcc
Lattice constants (a, c), Å	2·27, 3·59	4·04	2·86
Melting point, °C	1285	660	1540
Boiling point, °C	2507–2970	2057	2735
Specific heat, cal/g°C	0·475 (30°C)	0·214 (20°C)	0·108 (20°C)
Thermal conductivity, cal/cm cm^2 °C	0·385	0·52	0·189
Thermal expansion coefficient, 10^{-6}/°C	11·6	22·9	11·7
Electrical resistivity (20°C), μohm/cm	5·88	2·83	10·0
Modulus of elasticity, 10^6 psi	36–44	10·3	28·5
Ultimate tensile strength, 10^3 psi	33–51	13	—
Hardness (Brinell)	60–125	25	60
Mass absorption coefficient, cm^2/g			
for 14·4 keV gamma rays	0·24	8·0	>50
for 23·8 keV gamma rays	0·18	1·1	8·2

heat-transfer coefficient around the profile of a blade in cascade using surface heating strips [39]. It has been shown, however, that heat transfer to a blade of an operating turbine is higher than for a blade in a stationary cascade, due to steam turbulence following combustion, centrifugal effect on the boundary layer, and secondary flow engendered by non-uniform conditions in the flow approaching the turbine blades [40, 41]. For this reason Walker and Markland have measured the coefficients of secondary flow and heat transfer, but again at the surface of a stationary blade in a cascade [41].

It must be observed that these experiments were done, even if great care was taken to reproduce exactly an operating turbine, in stationary conditions and on specially-built devices; the values obtained for heat-transfer coefficients in these conditions are different from those existing in an actual rotating turbine. Moreover, when all the heat-transfer coefficients are known, the thermal gradients within a blade can be evaluated only for simplified blade models. In the majority of cases, this evaluation is difficult, if not impossible, to perform because either the blade has a complex form or too many influences are present; moreover, very often a system of equations is obtained which is not resolvable [38].

In these conditions, it has been necessary to measure directly thermal gradients by inserting thermocouples in different positions within the blades of a specially built turbine [38].

If, on the other hand, the Mössbauer effect is used, it is possible to take measurements directly during turbine operation, for one or more blades of a cascade, of the thermal gradients orthogonal to the blade height. As will be seen later in the description of an experimental arrangement, no major modification is needed for the measurement of thermal fields within the blades.

Reference will be made in the following to the Mössbauer spectrometer or analyser schematic description shown in Fig. 1 and to the turbine drawing of Fig. 6. In this last figure, C is the detector and A is the moving source, embedded in the turbine shaft together with a deplacement transducer F (for example a ferroelectric ceramic as the one used by Kündig [42]); a triangular voltage is fed to F in order to obtain a linear deplacement versus time of the source. A thermocouple T measures its temperature, which appears in both equations (7) and (10). This disposition is required from the condition that source and absorber are locked one with another in order to avoid that radial vibration, shaft imperfections and the impossibility to collimate exceedingly the gamma-ray beam broaden too much the Mössbauer absorption linewidth [42, 43].

A particular attention must be devoted to the possible influence of blading vibration on the measurement of temperature. In fact, if the radial component of the velocity fields induced by vibrations are comparable or higher than the relative velocity between source and blade caused by transducer F, some errors can be made on the temperature measurement because:

(a) the linewidth of the Mössbauer resonance absorption peak can be broadened exceedingly when blading vibration are not in phase with turbine rotation;

(b) the resonance absorption peak can shift when blading vibration produce a radial component of velocity in phase with turbine rotation; the resulting error can be very important because, as it is readily seen from equation (10), a variation of the velocity of the order of 6×10^{-5} cm/s (for ^{57}Fe) corresponds to an error of 1°C in the measured temperature.

However, an evaluation of the order of magnitude of the velocity radial-component induced by vibration can be made by considering the maximum amplitude these vibrations can reach in an actual turbine, as that shown in Fig. 6, revolving at 20000 rpm. In fact, blading vibrations can be of three different kinds: (i) radial, induced by centrifugal force variations caused by turbine angular velocity oscillations; it is readily seen that angular velocity changes of

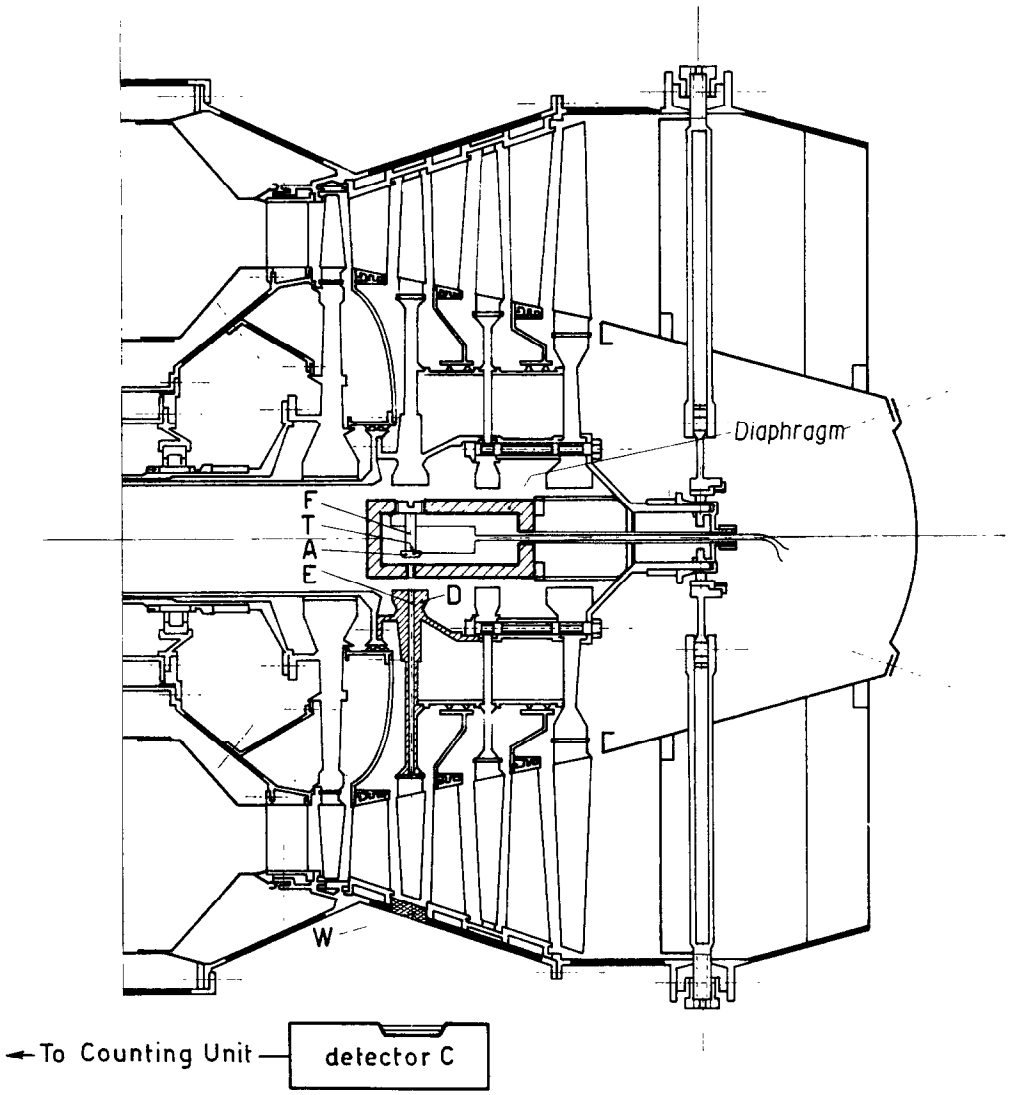

FIG. 6. Experimental set-up for thermal field measurement in turbine blading.

0·5 per cent can produce radial velocities of the order of 10^{-2} cm/s, which must be compared, for example, to the linewidth of the Mössbauer resonance for ^{57}Fe, $2{\cdot}7 \times 10^{-2}$ cm/s; (ii) bending, the radial component of velocity is an order of magnitude lower than the value obtained for the preceding case; and (iii) rolling, which produce no radial component of velocity.

It appears thus that, although a negligible linewidth broadening is to be expected, some resonance peak shifts are likely to occur (a velocity shift of 10^{-2} cm/s corresponds to an

error of about 200°C, in the case of ^{57}Fe) if the velocity oscillates in phase with the turbine rotating motion. However, it is possible to avoid this occurrence either by rotating the detector, during a reading, through 2π or by using several detectors placed around the turbine; moreover, this second arrangement reduces the time needed for a temperature measurement by allowing to detect gamma rays during the entire blade period of rotation.

To avoid absorption of gamma rays by the turbine structure, and by the disc that holds the blades is the principal problems to be faced when designing an actual experiment. To reduce the absorption to a level compatible with an available source activity, a canal E must be excavated in the disc D, and a beryllium window W placed in correspondence with the gamma-ray beam on the turbine structure; moreover, the blade under observation will have to be built in beryllium, in which the Mössbauer active isotopes chosen will have to be seeded.

It is expedient to build the entire disc under observation in beryllium, by working it with spark-erosion technology [44], being this technology well-established with beryllium [45] if some precaution is taken to avoid its toxicity.

As previously reported, every mean temperature value along a given direction is obtained analysing an ensemble of data relative to different values of the velocity between source and absorber.

This data is the product of the counting rate (counts/s) multiplied by the time interval during which the measurement is made.

When Mössbauer resonance absorption does not occur, this counting rate is given by:

$$C = 3{\cdot}7 \times 10^{10} \frac{1}{1+\alpha} \frac{Afa\eta}{4\pi r^2} e^{-\mu x}, \quad \text{counts/s} \tag{13}$$

where A is the source activity (Curie). $1/(1+\alpha)$ is the internal conversion coefficient, f is the Mössbauer-events fraction in the source, η the detector efficiency, a the resolving power (cm^2), r the length (cm) over which the resolving power a must be maintained, μ the mass absorption coefficient (cm^2/g) for the material considered (without considering Mössbauer resonance absorption) and x the material thickness (g/cm^2)

Obviously, when Mössbauer absorption occurs, the counting rate C given by equation (13) is reduced, by a factor given by the product of the expression given in equation (7) and the number of active isotopes seeded in the material under observation. The amount of active isotopes to be seeded will then be determined by the requirement that the counting rate decreases by 30–40 per cent when resonance absorption occurs, in order to obtain an appreciable peak affected by sufficiently low error.

To have an order of magnitude for a possible experiment, a very rough estimate can easily be made with reference to a turbine such as shown in Fig. 6. If the source is 5 mC ^{57}Co, with $f = 0{\cdot}35$, 100 per cent detector efficiency, $a = 1$ mm^2 and $r = 3$ cm, $1/(1+\alpha) \simeq 10$, the counting rate will be:

$$c \simeq 160, \quad \text{counts/s}$$

if the length of the blade is 2·8 cm and the beryllium windows total a thickness of 0·2 cm.

At least 30 points, with a maximum relative error of 1 per cent are required to obtain a sufficiently approximate temperature value along a determined direction using the least-square analysis method suggested in section 4. Therefore, if a single datum of the ensemble (for each temperature value) is determined with 10000 counts [which gives a relative error of about 1 per cent, as shown in equation (12)] each temperature measurement will require about 0·5 h.

If the turbine is rotating, several detectors placed circularly around the turbine are necessary to allow the counting to proceed during the entire period of blade rotation around the turbine shaft.

This arrangement is required also, as previously said, in order to avoid the resonance peak

to shift owing to the presence of vibrations in phase with the rotation of the turbine blading.

Obviously, if the annulus of detectors is not complete, the time required for a reading would increase by a factor equal to the ratio of the angle covered by the detectors with respect to 2π.

Several errors are introduced, independently of the statistical error given by equation (12), when the turbine is rotating. Among them, the most important are those arising from the measurement of turbine frequency and source linear speed, those caused by vibrations, pressure, strain and thermal effects on linear motion transducer. Other errors can be due to the presence of magnetic fields, to the radiation background on the detector or can be originated by the electronic apparatus (gatewidth, electronic dead time). However, it has been shown [42, 43] that their contribution to the final errors affecting the Mössbauer absorption line parameters can be evaluated in actual experiments and is generally negligible.

6. CONCLUSION

The practical realization of the experiment proposed for the measurement of temperature within turbine blading presents undoubtly many problems to be faced and it is expensive. Nevertheless, as it yields a direct evaluation of thermal gradients existing inside the blade in normal operating conditions and without perturbing temperature fields, its interest can justify the complexity of the measuring device and arrangements.

On the other hand, it must be remembered that proposed application to turbine blading is a limiting case. In many other applications, the measurement of thermal gradients by using Mössbauer effect will be very easy although the results obtained could maintain all their importance. As a first example, the case of the measurement of thermal gradients within a partially filled solid annulus, briefly described at the end of section 4, can be recalled here.

In conclusion, temperature measurement using Mössbauer effect appear as a unique method for the direct determination of thermal gradients within solid samples without any interaction between existing temperature fields and the measuring apparatus. In consideration of this fact, the authors' opinion is that this method could be applied profitably whenever difficulties are a minor argument as compared to the expected results.

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POSSIBILITÉ DE MESURE DES CHAMPS THERMIQUES EN EMPLOYANT L'APPLICATION DE L'EFFECT MÖSSBAUER AUX AILETTES DE TURBINE

Résumé—L'emploi de l'effet Mössbauer pour mesurer les gradients thermiques à l'intérieur d'échantillons solides est proposé et décrit. Les rayons gamma produits dans une source de Mössbauer sont absorbés par des noyaux actifs placés dans l'échantillon sous observation. Il est alors possible, en analysant les résultats relatifs à l'intensité d'absorption, d'obtenir les valeurs de la fraction sans recul dépendant fortement de la température et du glissement de Döppler du second ordre. La connaissance des valeurs de la température moyenne le long de la direction de pénétration peut être obtenue avec une bonne précision et alors, en répétant la mesure, les gradients thermiques à l'intérieur de l'échantillon.

En outre, l'application de cette méthode à l'étude des champs thermiques dans les ailettes de turbine est proposés, et l'on donne quelques estimations grossières de l'intensité de la source, de la puissance de résolution et des intervalles de temps nécessaires.

ÜBER DIE MÖGLICHKEIT THERMISCHE FELDER, SPEZIELL IN TURBINENSCHAUFELN MITTELS DES MÖSSBAUEREFFEKTES ZU MESSEN

Zusammenfassung—Die Messung von Temperaturgradienten im Innern von festen Körpern durch den Mössbauereffekt wird vorgeschlagen und beschrieben. Die Gammastrahlen einer Mössbauerquelle werden durch aktive Kerne, mit denen die betrachtete Probe dotiert ist, absorbiert. Durch Analyse der Daten im Bezug auf die Absorptionsintensität ist es dann möglich, sowohl die Werte des stark temperaturabhängigen rückstossfreien Tiels, als auch die der Doppelverschiebung zweiter Ordnung zu erhalten. Die mittleren Temperaturwerte längs der Durchtrittsrichtung kann man mit angemessener Genauigkeit ermitteln und dann durch Wiederholen der Messung, die Temperaturgradienten im Innern der Probe.

Darüberhinaus wird die Anwendung dieser Methode auf das Studium von Temperaturfeldern in Turbinenschaufeln vorgeschlagen und es werden einige rohe Abschätzungen der Intensität der Quelle, des Auflösungsvermögens und der benötigten Zeitintervalle gegeben.

О ВОЗМОЖНОСТИ ИЗМЕРЕНИЯ ТЕМПЕРАТУРНЫХ ПОЛЕЙ ЛОПАТОК ТУРБИНЫ, ИСПОЛЬЗУЯ ЭФФЕКТ МЕССБАУЭРА

Аннотация—Предлагается и описывается применение эффекта Мёссбауэра для измерения температурных градиентов в твёрдых образцах. Гамма-лучи, выходящие из источника Мёссбауэра, поглощаются активными ядрами, находящимися в наблюдаемом образце. Таким образом, анализируя данные по результирующей интенсивности излучения, можно получить значение интенсивности поглощения телом в зависимости от температуры и величину допплеровского смещения второго порядка. Можно с большой точностью получить распределение температуры по направлению проникновения излучения и затем путём повторных измерений определить температурные градиенты в образце.

Далее предлагается использовать этот метод для изучения температурных полей в лопатках, и приводятся грубые оценки интенсивности источника, разрешающей способности и необходимых промежутков времени.